# High-Resolution Electron Microscopy of Microsyntactic Intergrowth in $\mathbf{V}_{n} \mathbf{O}_{\mathbf{2 n - 1}}$ 

Y. HIROTSU,* Y. TSUNASHIMA, and S. NAGA KURA<br>Department of Metallurgy, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo, Japan<br>and H. KUWAMOTO $\dagger$ and H. SATO<br>School of Materials Engineering, Purdue University, West Lafayette, Indiana 47907

Received December 3, 1981


#### Abstract

Structural features of the Magnéli phases $\mathrm{V}_{n} \mathrm{O}_{2 n-1}$ were studied by means of high-resolution transmission electron microscopy. Microsyntactic intergrowth on a unit cell scale of several phases with neighboring $n$ were frequently observed in arc-melted specimens, and this was ascribed to the structural compensation of the local composition fluctuation. Also observed were periodic microsyntaxy, where two neighboring phases intergrow periodically, and the crystallographic shear (CS) structure with laterally displaced CS planes. The origin of this microsyntactic intergrowth is discussed from the standpoint of the $C S$ plane interaction.


## Introduction

Among many transition metal oxides, a series of homologous long-period compounds is often observed between two stoichiometric phases with different compositions. Most of them take a layered-type structure or the crystallographic shear (CS) structure. They have the following characteristics: (1) They have the same structural symmetry and are constructed from the same structural building units forming oneor two-dimensional long-period structures. (2) The energy of the coherent boundary between two neighboring phases is sup-

[^0]posed to be negligibly small, when they coexist. (3) The thermodynamic stability is almost equal for the two neighboring phases, especially when the period is long.

Because of the above characteristics, the neighboring line phases can easily intergrow in narrow bands with a crystallographically common axis. Such an intergrowth has been observed by several authors by means of the lattice imaging technique in electron microscopy in hexagonal ferrites $(1,2)$ and in Magnéli phase compounds $\mathrm{V}_{n} \mathrm{O}_{2 n-1}(3-6)$. It was found that the neighboring phases frequently intergrow in narrow bands on a unit cell scale with common basal plane (hexagonal ferrites) or with common $C S$ plane ( $\mathrm{V}_{n} \mathrm{O}_{2 n-1}$ ). This type of syntactic intergrowth has been termed microsyntactic intergrowth (7).

In the present study, detailed high-reso-
lution lattice image observations were made for $\mathrm{V}_{n} \mathrm{O}_{2 n-1}(n=6-8)$ in order to establish the physical origin of microsyntaxy. Computer simulations of images were also made for the interpretation of image contrast.

## Experimental Procedures

A polycrystalline $\mathrm{V}_{n} \mathrm{O}_{2 n-1}$ specimen was prepared from a mixture of $\mathrm{V}_{2} \mathrm{O}_{3}$ and $\mathrm{V}_{2} \mathrm{O}_{5}$ by arc-melting. For the electron microscopic observation, it was ground into small particles. After being dispersed in acetone, the particles were collected on cop-per-supported holey carbon films. Selected area diffraction patterns showed that the specimen composition was in a range between $\mathrm{V}_{6} \mathrm{O}_{11}$ and $\mathrm{V}_{9} \mathrm{O}_{17}$.

Observations were made with an electron microscope of JEM-100C type operated at 100 kV . A high-resolution objective lens with a spherical aberration constant of 0.7 mm and a chromatic aberration constant of 1.05 mm was used.

Very thin crystal flakes which gave (010)* diffraction patterns were selected. In this case, the incident beam direction (along the $b$ axis of $V_{n} \mathrm{O}_{2 n-1}$ ) is parallel to the $C S$ plane and the spot density in the (010)* diffraction pattern is the highest among ( $h k l)^{* 1}$ patterns. The beam divergence was about $1 \times$ $10^{-3}$ rad. The multiple-beam imaging technique (8) was applied by the use of an objective lens aperture limiting the scattering angle within $2.1 \times 10^{-2} \mathrm{rad}$.

Two characteristic types of images related to the CS structure were frequently observed with relatively high image contrast. One is composed of closely spaced bright stripes with spacing of about $3 \AA$ which are obliquely intersected by dark lattice fringes having CS plane spacings. The other type gives rows of bright spots along the direction parallel to the $C S$ planes, and faint spotty or linear images exist between

[^1]these rows of bright spots. The image of the former type was usually observed under the condition of about $600 \AA$ under-focus, and the latter under about 900 to $1000 \AA$ underfocus. Most of the images were taken under these conditions and the obtained images were compared with computer-simulated images. In the image calculations, atomic positional parameters given by Horiuchi et al. (9) were used for $\mathrm{V}_{6} \mathrm{O}_{11}$ and $\mathrm{V}_{7} \mathrm{O}_{13}$. For $\mathrm{V}_{8} \mathrm{O}_{15}$, the parameters adopted were estimated from $\mathrm{V}_{6} \mathrm{O}_{11}$ and $\mathrm{V}_{7} \mathrm{O}_{13}$. The multiplebeam imaging theory (10,11) was applied and the spherical (12) and chromatic (13) aberrations of the electron optical system were taken into account, as well as the beam divergence (14). The thickness for one slice was $3.5 \AA(=b / 2)$. The absorption correction was made by adding an imaginary term to the projected potential. The number of the interacting beams used was 523,611 , and 693 for $\mathrm{V}_{6} \mathrm{O}_{11}, \mathrm{~V}_{7} \mathrm{O}_{13}$, and $\mathrm{V}_{8} \mathrm{O}_{15}$, respectively.

## Crystal Structures of $\mathbf{V}_{\boldsymbol{n}} \mathbf{O}_{\mathbf{2 n - 1}}$

The structure of $\mathrm{V}_{n} \mathrm{O}_{2 n-1}$ is well described as a periodic modulation of the rutile-type $\mathrm{VO}_{2}$ lattice by the $C S$ planes. Horiuchi et al. (9) have recently refined the crystal structures of $\mathrm{V}_{n} \mathrm{O}_{2 n-1}$ by single crystal X-ray analysis, especially for $\mathrm{V}_{4} \mathrm{O}_{7}$ to $\mathrm{V}_{7} \mathrm{O}_{13}$. Single-crystal X-ray structural analysis has also be done by Marezio et al. for $\mathrm{V}_{4} \mathrm{O}_{7}$ and $\mathrm{V}_{5} \mathrm{O}_{9}(15,16)$. These results revealed that the arrangement of $\mathrm{VO}_{6}$ octahedra is almost identical to the ideal one derived from the parent rutile-type structure by introducing the CS planes periodically (17), although the $\mathrm{VO}_{6}$ octahedra at the $C S$ plane are distorted considerably.

Figure 1 shows a perspective view of the $\mathrm{V}_{7} \mathrm{O}_{13}$ structure (space group $P \overline{1}$ ) showing the arrangements of $\mathrm{VO}_{6}$ octahedra. The triclinic axes are drawn after Horiuchi et al. (9). For $\mathrm{V}_{n} \mathrm{O}_{2 n-1}$ structures, relations between the basic translation vectors of the


Fig. 1. Perspective view of $\mathrm{V}_{7} \mathrm{O}_{13}$. The triclinic $\mathrm{V}_{7} \mathrm{O}_{13}$ structure represented by the arrangement of $\mathrm{VO}_{6}$ octahedra is viewed along the $b$ axis. The crystallographic shear planes are along the ( $a, b$ ) plane and the positions are indicated by arrows. Straight chains of the edge-sharing $\mathrm{VO}_{6}$ octahedra are extending along the $c$ axis of the rutile-type $\mathrm{VO}_{2}$ structure (along $\mathrm{c}_{\mathrm{r}}$ ). One unit cell includes $14 \mathrm{VO}_{6}$ octahedra, and the numbers $1,3,5$, and 7 are given to the $\mathrm{VO}_{6}$ octahedra having the central vanadium atoms at $y=\frac{1}{2}$, and $2,4,6$, and 8 to those having central vanadium at $y=0$. The octahedra with even and odd numbers along the $b$ axis share their corners. The dimensional parameters of the unit cell are: $a=5.439, b=7.005$, and $c=35.516 \AA . \alpha=40.9, \beta=72.6$, and $\gamma=109.0^{\circ}$. Space group $P \overline{1}$.
triclinic $\mathrm{V}_{n} \mathrm{O}_{2 n-1}$ lattice $\mathbf{a}_{n}, \mathbf{b}_{n}, \mathbf{c}_{n}$ and those of the rutile-type tetragonal $\mathrm{VO}_{2}$ lattice $\mathbf{a}_{\mathrm{r}}$, $\mathbf{b}_{\mathrm{r}}, \mathbf{c}_{\mathrm{r}}$ are $\mathbf{a}_{n}=-\mathbf{a}_{\mathrm{r}}+\mathbf{c}_{\mathrm{r}}, \mathbf{b}_{n}=\mathbf{a}_{\mathrm{r}}+\mathbf{b}_{\mathrm{r}}+\mathbf{c}_{\mathrm{r}}$, and $\mathbf{c}_{n}=(2 n-1)\left(\mathbf{b}_{\mathbf{r}}+\mathbf{c}_{\mathbf{r}}\right) / 2$ (9). Therefore, the $C S$ plane $(1 \overline{2} 1)_{\mathrm{r}}$ in the rutile-type lattice corresponds to $(001)$ for the triclinic $\mathrm{V}_{n} \mathrm{O}_{2 n-1}$ lattice. In the $\mathrm{V}_{7} \mathrm{O}_{13}$ structure, the (001) CS plane exists at every thirteen oxygen layers parallel to (001). In Fig. 1, the edge- and face-shared $\mathrm{VO}_{6}$ octahedra are seen in the area of the CS plane, while in the slab of $\mathrm{VO}_{2}$ structure the corner- and edge-shared $\mathrm{VO}_{6}$ octahedra are seen. The rows of the edge-shared $\mathrm{VO}_{6}$ octahedra in the $\mathrm{VO}_{2}$ slab are along the $\mathbf{c}_{\mathrm{r}}$ direction. In the ideal $C S$ structure, as drawn in Fig. 1, the CS plane can be formed by introducing the $[011]_{r} / 2$ shear to the $\mathrm{VO}_{2}$ lattice.

The CS plane spacing in $\mathrm{V}_{n} \mathrm{O}_{2 n-1}, d_{001}$, is
TABLE I
Interplaner Spacings of CS Planes $d_{001}$

| $\mathrm{V}_{n} \mathrm{O}_{2 n-1}$ | $\mathrm{~V}_{4} \mathrm{O}_{7}$ | $\mathrm{~V}_{5} \mathrm{O}_{9}$ | $\mathrm{~V}_{6} \mathrm{O}_{11}$ | $\mathrm{~V}_{7} \mathrm{O}_{13}$ | $\mathrm{~V}_{8} \mathrm{O}_{15}$ | $\mathrm{~V}_{9} \mathrm{O}_{17}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $d_{001}(\AA)$ | 6.052 | 7.721 | 9.386 | 11.053 | 12.704 | $14.365^{a}$ |

[^2]shown in Table I in relation to the $n$ value, which is calculated from the crystallographic data $(9,18)$.

## Results

## a. Microsyntactic Behavior

The intergrown structures composed of $\mathrm{V}_{n} \mathrm{O}_{2 n-1}$ and its neighboring phase $\mathrm{V}_{n^{\prime}} \mathrm{O}_{2 n^{\prime}-1}$ ( $n^{\prime}=n \pm 1$ ) were often observed to have the common (001) CS plane. The width of the neighboring phase was about a few hundred $\AA$. In such a case, the two-component phases gave overlapped diffraction patterns (3). However, the growth in the form of microsyntactic intergrowth was observed more frequently. In this case, the diffraction pattern from the matrix phase was accompanied by weak diffuse streaks passing through each diffraction spot in the [001]* reciprocal lattice direction. In most cases, only one neighboring phase was observed. In some cases, however, two neighboring phases were found to intergrow in the matrix; in other words, three phases coexist side by side microsyntactically in some areas. Figure 2a shows a selected area dif-


Fig. 2. (a) Selected area diffraction pattern (010)* from an area of microsyntactic intergrowth. A series of diffuse $10 l$ spots from intergrown $\mathrm{V}_{7} \mathrm{O}_{13}$ structure (down arrows) are seen in the matrix $\mathrm{V}_{8} \mathrm{O}_{15}$ diffraction pattern. (b) Multiple-beam lattice image of the area by the use of the diffraction spots shown in (a). Diffracted beams within the scattering angle of $2.1 \times 10^{-2}$ rad are contributing to the imaging. Rows of bright spots are parallel to the $C S$ planes. The regions $A, B$, and $C$ correspond to $\mathrm{V}_{8} \mathrm{O}_{15}$, $\mathrm{V}_{7} \mathrm{O}_{13}$, and $\mathrm{V}_{6} \mathrm{O}_{11}$ structural regions, respectively. The unit cell of $\mathrm{V}_{8} \mathrm{O}_{15}$ in $A$ is shown by the parallelogram. The picture was taken under about $1000-\AA$ under-focus condition.
fraction pattern from an area including three such phases. The reflection spots are somewhat diffuse and several extrareflection spots are seen (downward arrows) besides the series of $10 l$ reflection spots from the $\mathrm{V}_{8} \mathrm{O}_{15}$ matrix (upward arrows). These extra-reflections were identified to be those from the microsyntactically intergrown $\mathrm{V}_{7} \mathrm{O}_{13}$ phase. No other phases can be identified in the diffraction pattern. Figure 2 b is the corresponding multiple-beam lattice image. The amount of defocusing was about $1000 \AA$ (under focus). In the image, the rows of bright spots parallel to the (001) CS planes are seen, which have three different spacings. The regions having these three different spacings are distinguished in the figure by the letters A , B, and C. It was found that the narrower spacings represented by the letters B and C correspond to those of the (001) CS planes of $\mathrm{V}_{7} \mathrm{O}_{13}$ and $\mathrm{V}_{6} \mathrm{O}_{11}$ structure, respectively. Figures 3 a , b , and c are computer-simulated images of $\mathrm{V}_{8} \mathrm{O}_{15}, \mathrm{~V}_{7} \mathrm{O}_{13}$, and $\mathrm{V}_{6} \mathrm{O}_{11}$ structures based on the $1000-\AA$ under-focus condition and the crystal thickness of $70 \AA$.

The arrangements of the rows of bright spots in the simulated images are in good agreement with those in the observed images. In the simulation, the images between the rows of bright spots vary according to the structures, but in the observed images these features are not clear. The image simulation shows that the rows of bright spots are exactly on the line of the projection of the $C S$ plane.
From the above results, it can be said that the $\mathrm{V}_{n} \mathrm{O}_{2 n-1}$ phases are thermodynamically definable down to the unit cell scale width. Since the thermodynamic stabilities of these phases are almost equal (19) and the formation energy of the CS plane is thought to be smaller to that of the assembly of point defects (oxygen vacancies) for the compensation of the nonstoichiometry (20), various phases can coexist especially when the specimens are prepared under nonequilibrium conditions.

## b. Periodic Microsyntaxy

In addition to the ordinary microsyntactic intergrowth in which intergrown phase


FIG. 3. Calculated images of (a) $\mathrm{V}_{8} \mathrm{O}_{15}$, (b) $\mathrm{V}_{7} \mathrm{O}_{13}$, and (c) $\mathrm{V}_{6} \mathrm{O}_{11}$ structure. Amount of defocus $\Delta f=$ $1000 \AA$. Crystal thickness $D=70 \AA$. The beam incidence direction is inclined from the $b$ axis by an amount of $3 \times 10^{-3} \mathrm{rad}$ to [001]*. The rows of bright spots are on the line of the projections of $C S$ planes.
appears in a random fashion, as shown in Fig. 2b, a periodic intergrowth of two neighboring phases is often found. This type of intergrowth was first found in a $\mathrm{V}_{6} \mathrm{O}_{11}$ specimen prepared by a vapor trans-
port method and was termed periodic microsyntaxy, where $\mathrm{V}_{6} \mathrm{O}_{11}$ and $\mathrm{V}_{7} \mathrm{O}_{13}$ intergrew alternatively on a unit cell scale to form $\mathrm{V}_{13} \mathrm{O}_{24}$ structure (4).
Figure 4 shows an example of another


Fig. 4. High-resolution multiple-beam image showing the periodic microsyntactic intergrowth. Beam is incident along the $b$ axis. The image was taken by using the diffraction spots inside the encircled area (scattering angle: $2.1 \times 10^{-2} \mathrm{rad}$ ) in the attached diffraction pattern. The focusing condition is about $600 \AA$ under-focus.
type of periodic microsyntaxy found in the present arc-melted specimen. The micrograph was taken under an about $600-\AA$ un-der-focus condition, which is close to the optimum-focus condition ( $620 \AA$ underfocus) for the structure image (8). In the figure, a periodically modulated structure is seen with boundaries parallel to the (001) $C S$ plane in the matrix $\mathrm{V}_{8} \mathrm{O}_{15}$. From the period of the dark lattice fringes parallel to the (001) plane and the fine dark stripes running obliquely to the dark fringes, the unit of the periodically intergrown structure was found to be composed of double $\mathrm{V}_{8} \mathrm{O}_{15}$ and single $\mathrm{V}_{7} \mathrm{O}_{13}$ unit cells with the common $C S$ planes. Therefore, $\mathrm{V}_{23} \mathrm{O}_{43}$ can be assigned for the periodically intergrown structure.

Since the image was taken under nearly optimum focusing condition, it provides information on the atomic arrangements. Judging from the $\mathrm{V}_{7} \mathrm{O}_{13}$ structure shown in Fig. 1, the dark lattice fringes in Fig. 4 may reflect the ( 001 ) $C S$ planes and the fine dark stripes may also reflect the rows of $\mathrm{VO}_{6}$ octahedra intersecting the (001) CS planes aslant. Structure factor calculations for the $h 0 l$ reflections of $\mathrm{V}_{7} \mathrm{O}_{13}$ and $\mathrm{V}_{8} \mathrm{O}_{15}$ showed that the structure factor coming from vanadium atoms, $F_{h 0 l}^{\mathrm{V}}$, is always much larger than that from oxygen atoms, $F_{\text {hol }}$. The ratio of $F_{h o l}^{\mathrm{y}} / F_{h o l}^{\mathrm{g}}$ ranges from about 5 to 400 depending on the indices of $h 0 l$. Therefore, the multiple-beam lattice image is mainly contributed by vanadium atoms. Figure 5 shows the atomic arrangement of vanadium atoms in $\mathrm{V}_{7} \mathrm{O}_{13}$ viewed along the $b$ axis, where they are represented by large ( $\mathrm{V}_{\text {odd }}$ ) and small $\left(\mathrm{V}_{\text {even }}\right)$ open circles and their distance is $3.5 \AA(=b / 2)$ along the $b$ axis. These chains of vanadium atoms along the $b$ axis are arranged along the $\mathbf{c}_{\mathrm{r}}^{\prime}\left(=-6.5 \mathbf{a}^{\prime}+\right.$ $c^{\prime}$ ) direction, ${ }^{2}$ which makes an angle of $38^{\circ}$

[^3]

Fig. 5. Atomic arrangements of vanadium atoms in the $\mathrm{V}_{7} \mathrm{O}_{13}$ structure viewed along the $b$ axis. The atomic rows along the $c_{r}^{\prime}=-6.5 a^{\prime}+c^{\prime}$ direction are shifted at the CS plane. The parallelogram indicates the $b$ projection of the unit cell. Vanadium atoms indicated by letters $V_{1}, V_{3}, V_{5}$, and $V_{7}$ are separated from those indicated by $V_{2}, V_{4}, V_{6}$, and $V_{8}$ by $b / 2$.
to the CS plane. The spacing between these vanadium atom rows normal to the $\mathbf{c}_{\mathbf{r}}^{\prime}$ direction is $3.1 \AA$. For $\mathrm{V}_{8} \mathrm{O}_{15}$, the structural description is the same if we add another vanadium layer along the (001) CS plane and consider the rows of vanadium atoms in the $\mathbf{c}_{\mathrm{r}}^{\prime}=-7.5 \mathbf{a}^{\prime}+\mathbf{c}^{\prime}$ direction. Because of the existence of $C S$ planes, the rows of vanadium atoms in the $\mathbf{c}_{r}^{\prime}$ direction are kinked at the CS plane. On the other hand, the observed spacing between the periodically arranged dark fine stripes is about $2.9 \AA$ and the stripes make an angle of about $40^{\circ}$ with the dark linear fringes. Thus the observed dark fine stripes may correspond to the rows of vanadium atoms in the $\mathbf{c}_{\mathrm{r}}^{\prime}$ direction, since according to the electron optical theory of lattice imaging for thin crystal under the optimum-focus condition (2I), the image intensity is proportional to the projection of the crystal potential. In order to confirm this conclusion, image simulations were made for $\mathrm{V}_{7} \mathrm{O}_{13}$ and $\mathrm{V}_{8} \mathrm{O}_{15}$ structures under the same imaging conditions as in the experiment. Figures 6 a and b show the calculated images corresponding to the crystal thickness of $70 \AA$. The dark stripes in the simulated images are in good agreement


Fig. 6. Calculated images of $\mathrm{V}_{7} \mathrm{O}_{13}$ (a) and $\mathrm{V}_{8} \mathrm{O}_{15}$ (b). $\Delta f=600 \AA, D=70 \AA$. The beam incidence is exactly parallel to the $b$ axis. Bright lines in the calculated image of $\mathrm{V}_{7} \mathrm{O}_{13}$ correspond well to the potential valleys of vanadium atoms found in Fig. 5.
with the observed ones with respect to both the spacing and the direction. An examination of the positions of vanadium atoms in the simulated images showed that the bright stripes correspond to the potential valleys between the atomic rows of vanadium. It was also confirmed that the model of $\mathrm{V}_{23} \mathrm{O}_{43}$ formed by the periodic intergrowth of $\mathrm{V}_{7} \mathrm{O}_{13}$ and $\mathrm{V}_{8} \mathrm{O}_{15}$ is correct and that the dark lattice fringes intersecting the dark fine stripes really represent the $C S$ planes with
the spacings of $11.1 \AA\left(\mathrm{~V}_{7}^{\prime} \mathrm{O}_{13}\right)$ and $12.7 \AA$ $\left(\mathrm{V}_{8} \mathrm{O}_{15}\right)$, respectively.

## c. CS Planes with Steps

According to the present lattice image observations of the CS structures, most of the images of $C S$ planes are straight and parallel to each other. In some cases, however, local variations in the CS plane spacings ( $C S$ planes with steps) have been observed. Figure 7 shows an example of such


Fig. 7. A multiple-beam lattice image of the structure mainly composed of $\mathrm{V}_{6} \mathrm{O}_{11}$ and $\mathrm{V}_{7} \mathrm{O}_{13}$. Steps in the $C S$ planes are observed in the encircled areas.
images. The structure is composed of a mixture of $\mathrm{V}_{7} \mathrm{O}_{13}$ and $\mathrm{V}_{6} \mathrm{O}_{11}$ in most areas. The beam incidence is almost parallel to the common $b$ axis and the focusing condition is about $1000 \AA$ under-focus. At several sites, especially in the encircled areas $A$ and $B$, steps of the CS planes are observed. By image simulations of $\mathrm{V}_{6} \mathrm{O}_{11}, \mathrm{~V}_{7} \mathrm{O}_{13}$, and $\mathrm{V}_{8} \mathrm{O}_{15}$ structures, it was confirmed that the observed rows of bright spots are exactly on the lines of the projections of the CS planes. By comparing the results of image simulations with the observed images, it was concluded that a pair of $\mathrm{V}_{6} \mathrm{O}_{11}$ and $\mathrm{V}_{8} \mathrm{O}_{15}$ structural periods changes into the
period of $\mathrm{V}_{7} \mathrm{O}_{13}$ in the area A , while in B , $\mathrm{V}_{6} \mathrm{O}_{11}$ and $\mathrm{V}_{7} \mathrm{O}_{13}$ change their arrangements to form a periodic microsyntaxy producing the $\mathrm{V}_{13} \mathrm{O}_{24}$ structure. Therefore, the step structure in the $C S$ plane may represent a transitional stage to a regular $C S$ plane formation.

The calculated images of $\mathrm{V}_{8} \mathrm{O}_{11}, \mathrm{~V}_{7} \mathrm{O}_{13}$, and $\mathrm{V}_{8} \mathrm{O}_{15}$ under the same imaging condition are shown in Figs. 8a to c . The projections of the unit cells along the $b$ axis are represented by the parallelograms connecting the bright spots $i j k l$. The shape and the dimension of each parallelogram and also the image contrast around the CS plane change


Fig. 8. Calculated images of $\mathrm{V}_{8} \mathrm{O}_{11}$ (a), $\mathrm{V}_{7} \mathrm{O}_{13}$ (b), and $\mathrm{V}_{8} \mathrm{O}_{15}$ (c). $\Delta f=980 \AA, D=35 \AA$. The beam incidence is inclined from the $b$ axis by $3 \times 10^{-3} \mathrm{rad}$ toward $[001]^{*}$. In the figures, the rows of bright spots are on the line of the projections of CS planes, and the unit cells are shown by the parallelograms connecting $i j k l$.
depending on the structures of $\mathrm{V}_{6} \mathrm{O}_{11}$, $\mathrm{V}_{7} \mathrm{O}_{13}$, and $\mathrm{V}_{8} \mathrm{O}_{15}$.

In arc-melted specimens, which are rather quickly prepared, the thermal history may be different from place to place. Accordingly, it may be possible to observe the intermediate stage of the CS plane formation. This possibility will be discussed in the following section.

## Discussion

In the arc-melted specimen, the distribution of the microsyntactically intergrown phases is more random than that in specimens grown by the vapor transport technique. The degree of the composition fluctuation is considered to be higher in the arc-melted specimen. Because of the ease of the CS plane formation compared with the atomic vacancy formation, the local composition fluctuation is possibly compensated by the microsyntactic intergrowth of the neighboring phases. A high-resolution imaging work seems to be necessary when we examine the solid solubility range of a certain phase which belongs to a series of line phases, because even the selected area diffraction method often fails to detect the existence of the intergrowth of other phases on a unit cell scale as shown in Fig. 2. This kind of argument may become important with respect to the nonstoichiometry in the case of a layered compound like $\beta$-alumina (22-24).

By electron microscopic studies on the reduction process of $\mathrm{TiO}_{2}(25,26)$, (Ti, V) $\mathrm{O}_{2}$ (27), and $\mathrm{MoO}_{3}(28)$, it has been shown that randomly arranged $C S$ planes (Wadsley defects) change into bundles of CS planes during the course of reduction, and further into the ordered arrangement of CS planes of the Magnéli phase compounds. In order to explain these apparent ordering phenomena of the CS planes theoretically, some authors have attempted to estimate the change in the free energy of
the crystal by introducing elastic or electrostatic interaction potential between the $C S$ planes (29,30). According to their theoretical calculations, the elastic interaction seems to be predominant for the ordered $C S$ plane formation. If the reduction of the free energy by the formation of ordered structure is large enough, randomly arranged microsyntactically intergrown structures can change into the ordered structure by further annealing. The periodic microsyntaxy can be a result of such a structural relaxation.

In order to explain the formation of such periodic microsyntactic phases as $\mathrm{V}_{13} \mathrm{O}_{24}$ and $\mathrm{V}_{23} \mathrm{O}_{43}$, it may be desirable to introduce the concept of the reduction in the elastic free energy by the formation of periodically arranged microsyntactically intergrown structure. The reduction of the free energy can be understood qualitatively by the following simple argument. In Fig. 9, the average interplanar spacing between the neighboring oxygen layers parallel to the (001) $C S$ plane of the $\mathrm{V}_{n} \mathrm{O}_{2 n-1}$ is plotted against the $n$ value. The crystallographic data by Horiuchi et al. $(9,18)$ are used for the calculation. The plot at $n=\infty$ corresponds to the ( $1 \overline{2} 1$ ) interplanar spacing of rutile-type $\mathrm{VO}_{2}$ lattice. The decrease in the average interplanar spacing $d_{001} /(2 n-1)$ with $n$ can be interpreted as a result of an elastic repulsive interaction between the $C S$ planes. In


INTER-OKYGEN LAYER DISTANCE OF $V_{n} 0_{2 n-1}$
Fig. 9. Dependence of average interoxygen layer distance $d_{001} /(2 n-1)$ on $n$.
this case, for the mixture of $\mathrm{V}_{6} \mathrm{O}_{11}$ and $\mathrm{V}_{7} \mathrm{O}_{13}$, for example, a unit structure of $\mathrm{V}_{6} \mathrm{O}_{11}$ may exert a compressive stress to the averaged surrounding structure, while a unit structure of $\mathrm{V}_{7} \mathrm{O}_{13}$ may exert a tensile stress to it. Therefore, a periodic alternative formation of $\mathrm{V}_{6} \mathrm{O}_{11}$ and $\mathrm{V}_{7} \mathrm{O}_{13}$ unit structures to form $\mathrm{V}_{13} \mathrm{O}_{24}$ structure may reduce the strain energy involved in the crystal. This situation is similar to the case of ordered structure formation in a binary alloy whose constituent species are different in atomic size.

It is hard to suppose that a periodic concentration fluctuation on a unit cell scale is first created in the matrix and transforms into the periodic microsyntactic structure. A reasonable formation mechanism of the
periodic microsyntactic structure may be that an inhomogeneous $C S$ structure is first formed with various $C S$ plane spacings from the concentration fluctuation in the specimen and then the $C S$ planes rearrange to make the periodic microsyntactic structure. The image in area B in Fig. 7 is considered to show the transitional state toward the formation of the periodic microsyntaxy composed of $\mathrm{V}_{7} \mathrm{O}_{13}$ and $\mathrm{V}_{6} \mathrm{O}_{11}$. Similarly, a redistribution process of the $C S$ planes to form a homogeneous $C S$ structure from an inhomogeneous one, such as $\mathrm{V}_{6} \mathrm{O}_{11}+\mathrm{V}_{8} \mathrm{O}_{15}$ $\rightarrow 2 \mathrm{~V}_{7} \mathrm{O}_{13}$, may take place in order to reduce the strain energy of the system. The image in area A in Fig. 7 seems to show the transitional state of such a reaction.
Based on the observed CS planes with

(a)

(b)

Fig. 10. (a) Atomic arrangement of oxygen (open circle) and vanadium (solid circle) atoms viewed along the $a$ axis of the rutile-type $\mathrm{VO}_{2}$ lattice. This corresponds to the image in area A in Fig. 7, and represents the formation of $2 \mathrm{~V}_{7} \mathrm{O}_{13}$ from $\mathrm{V}_{8} \mathrm{O}_{11}$ and $\mathrm{V}_{8} \mathrm{O}_{15}$ by the kinking of the $C S$ plane. D indicates a vanadium vacancy. (b) A model showing the lateral motion of the $C S$ plane to form the structure in (a) by the cooperative jumps of vanadium atoms in the direction indicated by arrows. See that a vanadium vacancy is formed at the step of the CS plane.
steps, as shown in Fig. 7, a possible mechanism for the rearrangement of $C S$ planes is presented in the following. Figure 10a shows a structural model for the encircled area $A$ in Fig. 7 where $\mathrm{V}_{6} \mathrm{O}_{11}$ and $\mathrm{V}_{8} \mathrm{O}_{15}$ react to form double $\mathrm{V}_{7} \mathrm{O}_{13}$. The atomic arrangement on an idealized (100) r plane of the rutile-type $\mathrm{VO}_{2}$ structure, which corresponds to the ( $\overline{1} 10$ ) plane of $\mathrm{V}_{n} \mathrm{O}_{2 n-1}$, is depicted, together with the traces of intersecting CS planes (solid lines). The ( $1 \overline{2} 1)_{r}$ or the (001) $\mathrm{V}_{n} \mathrm{O}_{2 n-1} C S$ plane makes an angle of $110.9^{\circ}$ with the $(001)_{\mathrm{r}}$ plane. Oxygen atoms (open circles) are closely packed and vanadium atoms (solid circles) are on them. They make a double atomic layer and stack up in a way so that every double atomic layer is displaced successively by a vector [110] ${ }_{\mathrm{r}} / 2$ with respect to the next layer. A vanadium defect site (lettered by D) always exists at the step of the $C S$ plane between $\mathrm{V}_{6} \mathrm{O}_{11}$ and $\mathrm{V}_{8} \mathrm{O}_{15}$. An energetically most favorable model for the formation of this stepped structure is schematically shown in Fig. 10b, where the movement of vanadium atoms is cooperative along the $C S$ plane as indicated by arrows. This model is based on the migration mechanism of $C S$ planes proposed by Andersson and Wadsley (31). In this model, the step of the $C S$ plane can be regarded as a termination point of the cooperative movement of vanadium atoms or as an initiation point of it. In the former sense, the cooperative motion of vanadium atoms must occur one after another, for example, from vanadium 1 to 5 in Fig. 10b, while in the latter sense, the cooperative motion starts from vanadium 5 and propagates to the lower-numbered vanadium atoms. We may call the position at the stepped CS plane a nucleation site for the growth process of $\mathrm{V}_{7} \mathrm{O}_{13}$ structure, if the position is the initiation site for the cooperative movement of vanadium atoms. The nucleated $C S$ structure ( $\mathrm{V}_{7} \mathrm{O}_{13}$ in this case) can be grown along the direction parallel to the $C S$ plane by the cooperative metal atom jumps to
give the lateral movement of the $C S$ plane. The formation of the periodic microsyntactic structure is also possible by the present model of rearrangement of $C S$ plancs.

## References

1. Y. Hirotsu and H. Sato, J. Solid State Chem. 26, 1 (1978).
2. Y. Hirotsu, H. Sato, Y. C. Tang, and S. Nagakura, "Proceedings of the 3rd International Conference on Ferrites, Kyoto, 1980,'’ p. 345.
3. Y. Hirotsu, S. P. Faile, and H. Sato, Mater. Res. Bull. 13, 895 (1978).
4. Y. Hirotsu and H. Sato, Mater. Res. Bull. 15, 41 (1980).
5. H. Kuwamoto, N. Otsuka, and H. Sato, J. Solid State Chem. 36, 133 (1981).
6. J. R. Gannon and R. J. D. Tilley, J. Solid State Chem. 25, 301 (1978).
7. H. Sato and S. Shinozaki, Mater. Res. Bull. 9, 679 (1974).
8. J. M. Cowley and S. Lijma, Z. Naturforsch, A 27, 445 (1972).
9. H. Horiuchi, N. Morimoto, and M. Tokonami, J. Solid State Chem. 17, 407 (1976).
10. P. Goodman and A. F. Moodie, Acta Crystallogr. Sect. A 30, 280 (1974).
11. J. M. Cowley, "Diffraction Physics," NorthHolland, Amsterdam (1975).
12. O. Scherzer, J. Appl. Phys. 20, 20 (1949).
13. P. L. Fejes, Acta Crystallogr. Sect. A 33, 109 (1977).
14. J. Frank, Optik 38, 519 (1973).
15. M. Marezio, D. B. McWhan, P. D. Dernier, and J. P. Remeika, J. Solid State Chem. 6, 213 (1973).
16. M. Marezio, P. D. Dernier, D. B. McWhan, and S. Kachi, J. Solid State Chem. 11, 301 (1974).
17. S. Andersson and L. Jahnberg, Ark. Kemi 21, 412 (1963).
18. H. Horiuchi, M. Tokonami, N. Morimoto, K. Nagasawa, Y. Bando, and T. Takada, Mater. Res. Bull. 6, 833 (1971).
19. J. S. Anderson, "The Chemistry of Extended Defects in Non-Metallic Solids," p. 1, North-Holland, Amsterdam (1970).
20. C. R. A. Catlow, Modulated structures 1979, In "AIP Conference Proceedings No. 53, p. 149 (1979).
21. D. F. Lynch, A. F. Moodie, and M. A. O'Keefe, Acta Crystallogr. Sect. A 31, 300 (1975).
22. H. Sato and Y. Hirotsu, Mater. Res. Bull. 11, 1307 (1976).
23. D. J. M. Bevan, B. Hudson, and P. T. MoseLey, Mater. Res. Bull. 9, 679 (1974).
24. M. Harata, Mater. Res. Bull. 6, 461 (1971).
25. L. A. Bursill, B. G. Hyde, O. Terasaki, and D. Watanabe, Philos. Mag. 20, 347 (1969).
26. J. Van Landuyt and S. Amelinckx, Mater. Res. Bull. 5, 267 (1970).
27. K. Kosuge and S. Kachi, Chem. Scr. 8, 70 (1975).
28. L. A. Bursill, Proc. Roy. Soc. Ser. A 311, 267 (1969).
29. E. Iguchi and R. J. D. Tilley, J. Solid State Chem. 21, 49 (1977).
30. Y. Shimizu and E. Iguchi, Phys. Rev. B 17, 2505 (1978).
31. S. ANDERSSON AND A. D. Wadsley, Nature (London) 211, 581 (1969).

[^0]:    * Present address: Technological University of Nagaoka, Nagaoka, Niigata, Japan.
    $\dagger$ Present address: Rockwell International Science Center, Thousand Oaks, California 91360.

[^1]:    ${ }^{1}(h k l)^{*}$ and $[h k l]^{*}$ represent the plane and the direction in the reciprocal lattice, respectively.

[^2]:    ${ }^{a}$ Extrapolated from the values of $\mathrm{V}_{4} \mathrm{O}_{7}$ to $\mathrm{V}_{8} \mathrm{O}_{15}$.

[^3]:    ${ }^{2}$ This corresponds to $\mathbf{c}_{\mathrm{r}}^{\prime}=-(2 n-1) \mathbf{a}^{\prime} / 2+\mathbf{c}^{\prime}$, where $a^{\prime}$ and $c^{\prime}$ are the $a_{n}$ and $c_{n}$ projected on the plane normal to the $b$ axis.

